HIGHER MATHEMATICS MATHEMATICS		8719/07 9709/07
May/June 2003		
		1 hour 15 minutes
Additional materials:	Answer Booklet/Paper Graph paper List of Formulae (MF9)	

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

## This document consists of **3** printed pages and **1** blank page.

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- 1 A fair coin is tossed 5 times and the number of heads is recorded.
  - (i) The random variable X is the number of heads. State the mean and variance of X. [2]
  - (ii) The number of heads is doubled and denoted by the random variable *Y*. State the mean and variance of *Y*. [2]
- 2 Before attending a basketball course, a player found that 60% of his shots made a score. After attending the course the player claimed he had improved. In his next game he tried 12 shots and scored in 10 of them. Assuming shots to be independent, test this claim at the 10% significance level.
  [5]
- 3 A consumer group, interested in the mean fat content of a particular type of sausage, takes a random sample of 20 sausages and sends them away to be analysed. The percentage of fat in each sausage is as follows.

27 26 28 28 28 29 29 30 30 31 32 32 32 33 33 34 34 34 35 35

Assume that the percentage of fat is normally distributed with mean  $\mu$ , and that the standard deviation is known to be 3.

- (i) Calculate a 98% confidence interval for the population mean percentage of fat. [4]
- (ii) The manufacturer claims that the mean percentage of fat in sausages of this type is 30. Use your answer to part (i) to determine whether the consumer group should accept this claim. [2]
- 4 A random variable *X* has probability density function given by

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find 
$$P(X > 1.5)$$
. [2]

- (ii) Find the mean of X. [2]
- (iii) Find the median of *X*. [3]
- 5 Over a long period of time it is found that the time spent at cash withdrawal points follows a normal distribution with mean 2.1 minutes and standard deviation 0.9 minutes. A new system is tried out, to speed up the procedure. The null hypothesis is that the mean time spent is the same under the new system as previously. It is decided to reject the null hypothesis and accept that the new system is quicker if the mean withdrawal time from a random sample of 20 cash withdrawals is less than 1.7 minutes. Assume that, for the new system, the standard deviation is still 0.9 minutes, and the time spent still follows a normal distribution.
  - (i) Calculate the probability of a Type I error. [4]
  - (ii) If the mean withdrawal time under the new system is actually 1.5 minutes, calculate the probability of a Type II error. [4]

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- 6 Computer breakdowns occur randomly on average once every 48 hours of use.
  - (i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use. [3]
  - (ii) Find the probability that the number of breakdowns in one year (8760 hours) of use is more than 200.
  - (iii) Independently of the computer breaking down, the computer operator receives phone calls randomly on average twice in every 24-hour period. Find the probability that the total number of phone calls and computer breakdowns in a 60-hour period is exactly 4. [3]
- 7 Machine *A* fills bags of fertiliser so that their weights follow a normal distribution with mean 20.05 kg and standard deviation 0.15 kg. Machine *B* fills bags of fertiliser so that their weights follow a normal distribution with mean 20.05 kg and standard deviation 0.27 kg.
  - (i) Find the probability that the total weight of a random sample of 20 bags filled by machine A is at least 2 kg more than the total weight of a random sample of 20 bags filled by machine B. [6]
  - (ii) A random sample of *n* bags filled by machine *A* is taken. The probability that the sample mean weight of the bags is greater than 20.07 kg is denoted by *p*. Find the value of *n*, given that p = 0.0250 correct to 4 decimal places. [4]

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